Performance of BCH and Convolutional Codes in Direct Sequence Spread Spectrum Packet Radio Networks

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ABSTRACT

The performance of forward error correction codes has been well documented in the open literature when the noise is additive, white and Gaussian. Little or no work exists on the performance of forward error correction codes when the noise is due primarily to multiple access interference. In this paper, we examine the performance of BCH and convolutional codes in a direct-sequence spread spectrum packet radio network. Packet errors are caused by a combination of noise at the receiver and interference between packet transmissions which overlap in time. The interference between packet transmissions produces dependent symbol errors at the output of the demodulator. In our work, we compute first an upper bound on the symbol error probability. Then we use this upper bound to calculate upper bounds on the packet error probability for both BCH and convolutional codes. Our results enable us to compare the performance of BCH and convolutional codes in the presence of multiple access interference.

1. Introduction

An attribute of spread spectrum signalling is its multiple access capability [1]. The most important indicator of the multiple access capability of a packet radio network is the induced packet probability.

The problem of computing packet error probabilities in direct-sequence spread spectrum (DS-SS) packet radio networks is difficult. Packet errors are caused by a combination of noise at the receivers and interference between packet transmissions, which overlap in time. The interference between packet transmissions produces dependent errors at the output of the demodulator. A lot of work has been directed towards the evaluation of the bit error probability in direct-sequence spread spectrum networks ([2], [3]). The dependency of the bit errors does not allow us to extend the results in [2] and [3], in order to compute the packet error probability.

In our work, we are going to examine the multiple access capability of a DS-SS packet radio network when BCH and convolutional codes are used for the encoding of the packets. It is worth noting that the performance of convolutional codes in a multiple access SS enviroment has been examined before ([4], equal power signals only). Furthermore, the performance of BCH codes in a multiple access SS environment has been dealt with in [5]. This paper uses the techniques developed in [4] and [5] to provide a full scale comparison between BCH and convolutional codes. Our numerical results indicate that BCH codes outperform convolutional codes, in a multiple access DS spread spectrum environment. Additional work is needed though to make more meaningful conclusions, since the numerical results correspond to upper bounds on the packet error probabilities.

2. The Model - Preliminaries

The model for direct sequence spread spectrum transmission considered here is described in [6]. The only difference is that the signature sequence is assumed to be sequence of independent, identically distributed, binary random variables (called chips), each equally likely to be +1 or -1. Each transmitter in the network has such a sequence, and each sequence is assumed to be independent of the sequences of other transmitters.

Let us assume that we have a slotted channel, K ($K>1$) packet transmissions occur within a slot, and a receiver locks on to packet #1. Our objective is to compute the probability that the receiver decodes packet #1 incorrectly. We denote this probability by $P_e(K)$.

The receiver is assumed to be a correlation receiver. The output of the recceiver corresponding to the mth bit ($0 \leq m \leq M-1$) of packet #1, is the random variable (see [1] for details)

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$$Z_m = n_m + (2^{-1}P_1)^{1/2}T\{b_m^{(1)} + \sum_{i=2}^{K}(P_i/P_1)^{1/2}I_{i,1}^m(\underline{b}_i^m, t_i, \theta_i)\} \quad (1)$$

Each n_m is a Gaussian random variable with zero mean and variance $N_oT/4$, where $N_o/2$ is the two sided spectral density of the white Gaussian noise and T is the data bit duration. The random variables n_m $(0\le m\le M-1)$ are independent. The variable $b_m^{(1)}$ represents a pair of consecutive data bits of packet #1. In particular, $\underline{b}_i^m = (b_{m-1}^{(i)}, b_m^{(i)})$, and each data bit $b_m^{(1)}$ is either +1 or −1. Each t_i or θ_i is a random variable representing the time delay (modulo T) or the phase angle (modulo 2π), respectively, of packet #i relative to packet #1. As in [4], we take the range of t_i to be the interval $(0,T]$ and the range of θ_i to be the interval $[0,2\pi)$. Finally, P_i is the power of packet #i at the receiver. The function $I_{i,1}^m$, which appears in (1), represents the normalized multiple access interference due to packet #i. It depends on $\underline{b}_i$, t_i, θ_i and the signature sequences corresponding to packets #i and #1 (see [4] for more details).

The detector decides that the mth bit of packet #1 is +1 or −1 if $Z_m>0$ or $Z_m<0$, respectively. It is easy to show that the mth bit of packet #1 is decoded correctly by the above detector if and only if the random variable

$$X_m = n_m^* + [1 + \sum_{i=2}^{K} b_m^{(1)} (P_i/P_1)^{1/2} I_{i,1}^m (\underline{b}_i^m, t_i, \theta_i)] \ ; \ 0\le m\le M-1 \quad (2)$$

is positive. In (2) each n_m^* is a Gaussian random variable with mean 0 and variance $N_o/2E_b$, where $E_b=P_1T$ is the energy per data bit of packet #1. The random variables n_m^* $(0\le m\le M-1)$ are statistically independent.

Let us now state two propositions.

<u>Proposition 1.</u> For the computation of $P_e(K)$ the t_i's $(2\le i\le K)$ need be known only to the nearest chip.

<u>Proposition 2</u> The packet error probability $P_e(K)$ is independent of the values of the data bit sequences.

The validity of propositions 1 and 2 is based on the fact that random signature sequences are utilized. An immediate consequence of propositions 1 and 2, is that the random variable X_m in (2) assumes the following equivalent form

$$X_m = n_m^* + [1 + \sum_{i=2}^{K} b_m^{(1)} (P_i/P_1)^{1/2}\{[a_{mN-1}^{(i)} a_{mN}^{(1)}] + [a_{mN}^{(i)} a_{mN}^{(1)} + a_{mN+1}^{(i)} a_{mN+1}^{(1)} + \ldots + a_{mN+N-1}^{(i)} a_{mN+N-1}^{(1)}] (1-t_i/T_c) + [a_{mN}^{(i)} a_{mN+1}^{(1)} + a_{mN+1}^{(i)} a_{mN+2}^{(1)} + \ldots + a_{mN+N-2}^{(i)} a_{mN+N-1}^{(1)}](1-t_i/T_c)\}\cos\theta_i/N; \ 0\le m\le M-1 \quad (3)$$

In (3) we assumed a rectangular chip waveform and N chips per bit; T_c is the chip duration, and $\{a_j^{(i)}\}$ is the signature sequence corresponding to packet #i.

The derivation of the upper bounds on the packet error probability $P_e(K)$, presented in the next section, will be based on the following assumption.

<u>Assumption 1.</u> Given the phase (θ_i) and the delay (t_i) of each interfering transmission $(2\le i\le K)$, the random variables X_m $(0\le m\le M-1)$ <u>are</u> independent.

In [5] a detailed discussion is conducted that justifies the adoption of the above assumption for the computation of upper bounds on the packet error probability induced in our system.

3. Upper bounds on the packet error probability.

Let us denote by p the symbol (bit) error probability induced in our system. Obviously,

$$p = \Pr(X_o < 0)$$

In [4] it is shown that for equal power signals at the receiver site the symbol (bit) error probability is upper bounded by the following expression:

$$q = Q[2E_b/N_o)^{1/2}] +$$

$$(1/\pi)\int_o^\infty u^{-1}\sin(u)\phi_2(u)[-\phi_1(u)]du \qquad (5)$$

where

$$\phi_1(u) = \{\cos(u/N)\}^{N(K-1)} \qquad (6)$$

$$\phi_2(u) = \exp[(-N_o/4E_b)u^2] \qquad (7)$$

and

$$Q(x) = (2\pi)^{-1/2}\int_o^\infty \exp(-u^2/2)\, du$$

Furthermore, in [5] a differenct upper bound (r) on the symbol (bit) error probability is derived. In particular, it is shown that

$$r = \inf_{z \geq 0}\{\exp(-z)E[\exp(zn_o^*)]$$

$$\prod_{i=2}^{K} E[\exp [P_i/P_1)^{1/2}J_i]\} \qquad (8)$$

with

$$J_i = [\sum_{j=0}^{N-1(i)} a_j/N];\ 2 \leq i \leq K \qquad (9)$$

The r bound on the symbol error probability is valid independently of whether signals arrive with equal or unequal power at the receiver site. However, for equal power signals, the q bound is tighter than the r bound.

Due to assumption 1, we can show that (see [5])

$$P_e(K) \leq \sum_{i=e+1}^{M} [\binom{M}{i} s^i(1-s)^{M-i}] = P_e^B(K) \quad (10)$$

for BCH codes. In (10), s corresponds to q for equal power signals and to r for unequal power signals.

Futhermore, for convolutional codes we can show that (see [4], [5])

$$P_e(K) \leq 1-[1-P_u(s)]^L = P_e^C(K) \qquad (11)$$

Where s corresponds to q for equal power signals and to r for unequal power signals. L is the number of information bits in the code. $P_u(s)$, the union bound, is difficult to evaluate exactly, and all analytical results are based on bounding $P_u(s)$ in terms of the transfer function T(D) of the code. The bound of Van de Meerberg [8] is used for the results presented here; it is given by

$$P_u(s) \leq \Gamma_{n_o}\{1/2[T(D) + T(-D)] +$$

$$1/2D\{T(D) - T(-D)]\}\Big|_{D=2s^{1/2}} \qquad (12)$$

where

$$\Gamma_{n_o} = \binom{2n_o-1}{n_o} 2^{-2n_o} \qquad (13)$$

and n_o is one half of the free distance of the code (or one half of the "free distance plus one" if the free distance is odd).

4. Numerical Results - Conclusions.

In Table 1, the upper bounds $P_e^B(K)$ and $P_e^C(K)$ on the packet error probability $P_e(K)$ are shown. Results are shown for the (63,30), (127,64), (255,131) and (1023,513) BCH codes. The best rate 1/2 constraint-length-7 binary convolutional codes are used. To make a fair comparison with the BCH codes we chose our convolutional codes to be approximately of the same length as our BCH codes (i.e. 60, 128, 262, 1026 length codes). The entries in Table 1 corresponds to different K choices, signal-to-noise ratio of 15dB, near far ratios of 0dB, 3dB, 6dB, and N = 127.

The numerical results show that the BCH codes outperform convolutional codes, at least for the bounds computed here. For shorter length codes the results are comparable. However, for larger length codes the comparison favors BCH codes. It is best to keep in mind that the performance of the convolutional codes is the result of three upper bounds for the bit error probability (p), the union bound (P_u), and the packet error probability (P_e). The calculation of the performance of the BCH codes is based on one upper bound for the bit error probability (p). Hence, at least part of the difference in performance may be due to looseness in the bounds computed for the convolutional codes. We are currently investigating the tightness of these bounds in an effort to make a more conclusive comparison regarding the performance of BCH and convolutional codes in DS-SS multiple access environment.

BIBLIOGRAPHY

[1] M.B. Pursley, "Spread Spectrum Multiple Access Communications," in Multi-User Communication Systems, editor G. Longo, Springer-Verlag; 1980.

[2] E.A. Geraniotis and M.G. Pursley, "Error Probability for Direct-Sequence Spread-Spectrum Multiple Access Communications-Part II: Approximations," *IEEE Transactions on Communications*, vol. COM-30, pp. 985-995, May, 1982.

[3] K. Yao, "Error Probability of Asynchronous Spread-Spectrum Multiple Access Communication Systems," *IEEE Transactions on Communications*, vol. COM-25, pp. 803-809, August, 1977.

[4] M.B. Pursley and D.J. Taipale, "Error Probabilities for Spread-Spectrum Packet Radio with Convolutional Codes and Viterbi Decoding," *IEEE Transactions on Communications*, vol. COM-35, pp. 1-12, January, 1987.

[5] M. Georgiopoulos, "Packet Error Probabilities in Direct Sequence Spread-Spectrum Packet Radio Network, with BCH Codes," MILCOM 88, San Diego, CA, pp. 15.3.1 - 15.3.5.

[6] J.S. Lehnert and M.B. Pursley, "Error Probabilities for Binary Direct-Sequence Spread-Spectrum Communications with Random Signature Sequences," *IEEE Transactions on Communications*, vol. COM-35, No.1, pp. 87-98, January, 1987.

[7] A.J. Viterbi, "Convolutional Codes and Their Performance in Communication Systems," *IEEE Transactions on Communications Technology*, vol. COM-19, pp. 751-772, October, 1971.

[8] L. Van de Meerberg, "A Tightened Upper Bounds on the Error Probability of Binary Convolutional Codes with Viterbi Decoding," *IEEE Transactions on Information Theory*, vol. IT-20, pp. 389-391, May, 1974.

[9] J. Storey and F. Tobagi, "Throughput Performance of a Direct Sequence CDMA Packet Radio Network," Computer Systems Laboratory, Stanford University, Stanford, CA, Rep. DEL TR 85-277, June, 1985.

TABLE 1

Upper Bounds on the Packet Error Probability $P_e(K)$

E_b/N_o = 15 dB,
N = 127,
Near-Far Ratio
= 3 dB

K=	M	$P_e^B(K)$	M	$P_e^C(K)$
04	63	2.97E-16	60	3.86E-13
	127	1.60E-23	128	8.24E-13
	255	2.83E-38	262	1.68E-12
	1023	9.12E-69	1026	6.60E-12
05	63	2.06E-11	60	1.19E-09
	127	6.25E-16	128	2.55E-09
	255	3.47E-25	262	5.23E-09
	1023	1.29E-66	1026	2.05E-08
06	63	3.06E-08	60	2.60E-07
	127	5.48E-11	128	5.54E-07
	255	1.04E-16	262	1.13E-06
	1023	4.24E-41	1026	4.44E-06
07	63	4.77E-06	60	1.28E-05
	127	1.29E-07	128	2.74E-05
	255	5.59E-11	262	5.61E-05
	1023	2.86E-24	1026	2.20E-04
08	63	1.73E-04	60	2.69E-04
	127	2.88E-05	128	5.74E-04
	255	4.55E-07	262	1.17E-03
	1023	3.11E-13	1026	4.59E-03
09	63	2.31E-03	60	3.72E-03
	127	1.24E-03	128	7.93E-03
	255	1.98E-04	262	1.61E-02
	1023	1.85E-06	1026	6.18E-02

TABLE 1

Upper Bounds on the Packet Error Probability $P_e(K)$

E_b/N_o = 15 dB,
N = 127,
Near-Far Ratio
= 0 dB

K=	M	$P_e^B(K)$	M	$P_e^C(K)$
07	63	2.70E-23	60	3.60E-18
	127	1.39E-34	128	7.69E-18
	255	2.26E-57	262	1.57E-17
	1023	0.00E+00	1026	6.17E-17
08	63	1.90E-20	60	3.93E-16
	127	4.15E-30	128	8.39E-16
	255	1.20E-49	262	1.71E-15
	1023	0.00E+00	1026	6.73E-15
09	63	3.70E-18	60	1.33E-14
	127	1.63E-26	128	2.84E-14
	255	1.94E-43	262	5.81E-14
	1023	0.00E+00	1026	2.27E-13
10	63	2.82E-16	60	3.86E-13
	127	1.47E-23	128	8.24E-13
	255	2.45E-38	262	1.68E-12
	1023	0.00E+00	1026	6.60E-12
11	63	1.05E-14	60	5.44E-12
	127	4.36E-21	128	1.16E-11
	255	4.50E-34	262	2.37E-11
	1023	0.00E+00	1026	9.31E-11
12	63	2.29E-13	60	5.18E-11
	127	5.44E-19	128	1.10E-10
	255	1.85E-30	262	2.26E-10
	1023	0.00E+00	1026	8.85E-10
13	63	3.23E-12	60	3.64E-10
	127	3.43E-17	128	7.78E-10
	255	2.34E-27	262	1.59E-09
	1023	3.42E-73	1026	6.23E-09
14	63	3.21E-11	60	2.02E-09
	127	1.25E-15	128	4.31E-09
	255	1.14E-24	262	8.82E-09
	1023	4.78E-65	1026	3.45E-09
15	63	2.40E-10	60	9.20E-09
	127	2.90E-14	128	1.96E-08
	255	2.54E-22	262	4.01E-08
	1023	5.96E-58	1026	1.57E-07

TABLE 1

Upper Bounds on the Packet Error Probability $P_e(K)$

E_b/N_o = 15 dB,
N = 127,
Near-Far Ratio
= 6 dB

K=	M	$P_e^B(K)$	M	$P_e^C(K)$
03	63	1.71E-11	60	1.04E-09
	127	4.67E-16	128	2.23E-09
	255	2.10E-25	262	4.57E-09
	1023	2.87E-67	1026	1.79E-08
04	63	4.35E-06	60	1.19E-05
	127	1.12E-07	128	2.55E-05
	255	4.42E-11	262	5.22E-05
	1023	1.46E-24	1026	2.04E-04
05	63	2.19E-03	60	3.50E-03
	127	1.16E-03	128	7.47E-03
	255	1.76E-04	262	1.52E-02
	1023	1.40E-06	1026	5.83E-02